

Integral Calculus

Integration – Introduction, Definition, Standard Formulae

Introduction to Integration

- Integration is a fundamental concept in calculus.
- It is often described as the **reverse process of differentiation**, but more broadly, it refers to finding the **area under curves**, among many other applications.

There are two main types of integration:

1. **Indefinite integration:** Represents a family of functions (no bounds)
2. **Definite integration:** Represents a numerical value (area under a curve between two points)

Definition of Indefinite Integration

- If you are given a function $f(x)$, then its **indefinite integral** is written as:

$$\int f(x) dx$$

- This represents the **antiderivative** of $f(x)$, which is another function $F(x)$ such that:

$$\frac{d}{dx}[F(x)] = f(x)$$

- We write:

$$\int f(x) dx = F(x) + C$$

- Where:
 - $F(x)$ is any one antiderivative of $f(x)$
 - C is the **constant of integration** (since differentiating a constant gives 0)

Standard Integration Formulae

- Here are some commonly used standard formulae:

1. Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1)$$

2. Exponential Functions:

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C (a > 0, a \neq 1)$$

3. Logarithmic Function:

$$\int \frac{1}{x} dx = \ln |x| + C$$

4. Trigonometric Functions:

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Definite Integrals

Definition

- A **definite integral** is the integral of a function between two specified limits.
- It gives the **net area** under the curve of the function between the lower limit a and upper limit b on the x -axis.
- It is denoted as:

$$\int_a^b f(x) dx$$

- Where:
 - $f(x)$ is the integrand.
 - a is the lower limit of integration.
 - b is the upper limit of integration.
 - dx indicates that the integration is with respect to variable x .

Fundamental Theorem of Calculus

- If $F(x)$ is the **indefinite integral** of $f(x)$, i.e.,

$$F(x) = \int f(x) dx$$

- Then:

$$\int_a^b f(x) dx = F(b) - F(a)$$

- So, you:
 1. Find the **indefinite integral** $F(x)$
 2. Evaluate it at the **upper limit** b
 3. Subtract the value at the **lower limit** a

Example

$$\int_0^2 x^2 dx$$

Step 1:

$$\int x^2 dx = \frac{x^3}{3}$$

Step 2:

- Evaluate from 0 to 2:

$$\frac{x^3}{3} \Big|_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$$

Properties of Definite Integrals

1. Reversal of limits:

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

2. Zero-width interval:

$$\int_a^a f(x) dx = 0$$

3. Additivity over intervals:

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

4. Linearity:

$$\int_a^b [af(x) + bg(x)] dx = a \int_a^b f(x) dx + b \int_a^b g(x) dx$$

- Definite integrals are especially important in **physics**, **engineering**, and **economics**, as they often represent total quantities over time, space, or other variables.

Method of Substitution

Purpose of Substitution

- The **substitution method** (also called **u-substitution**) is a powerful technique used to simplify complicated integrals by changing variables. It works similarly to the **chain rule** in differentiation, but in reverse.

Basic Idea

- Suppose you have an integral of the form:

$$\int f(g(x)) \cdot g'(x) dx$$

- Then, using substitution:

- Let $u = g(x)$

- Then $\frac{du}{dx} = g'(x) \Rightarrow du = g'(x) dx$

- This transforms the integral into:

$$\int f(u) du$$

- Which is typically easier to integrate.

Step-by-Step Process

- Choose a substitution:** Identify a part of the integrand that, when substituted, simplifies the expression.
- Compute du:** Differentiate your substitution to find du.
- Change all variables:** Rewrite the integral in terms of u and du.
- Integrate in u**
- Substitute back:** Replace u with the original expression.

Example

$$\int 2x \cdot \cos(x^2) dx$$

Step 1:

- Let $u = x^2 \Rightarrow du = 2x dx$
- Now the integral becomes:

$$\int \cos(u) du = \sin(u) + C$$

Step 2:

- Substitute back:

$$\sin(x^2) + C$$

Example

$$\int \frac{x}{x^2 + 1} dx$$

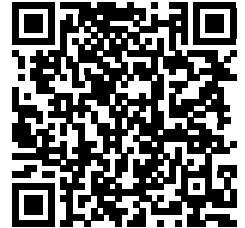


FREE PREVIEW

[Click here to visit our website](#)

PHARMACY PREMIUM NOTES

Complete subject
and semesters are
available on
FirstHope app only.



[Scan or click here](#)

Click here to get the app



Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram



[Scan or click here](#)

@FirstHopeNotes

Instagram

@firsthope.official

[Click here to follow >](#)

YouTube channel

FirstHope-Pharmacy / Live streams

[Click here to watch >](#)

WhatsApp channel

Follow FirstHope on WhatsApp

[Click here to join >](#)

FirstHope website

firsthopenotes.com

[Click here to visit >](#)

B.Pharm / D.Pharm / M.Pharm / Pharm.D

Preview notes. Complete subjects and semesters in the FirstHope app.