

MATHEMATICS • CLASS 11

CHAPTER 12

Limits and Derivatives

CHAPTER NOTES



CALCULUS: UNDERSTANDING CHANGE

Calculus helps us study how one quantity changes with another.
Newton and Leibniz developed calculus independently.

A falling body

The chapter models the distance fallen after a given time.
Distance is in metres and time is in seconds.

$$s(t) = 4.9t^2$$

Average velocity

Average velocity is the change in distance divided by the time taken.

$$v_{\text{avg}} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$s(0) = 0 \text{ m}; \quad s(1) = 4.9 \text{ m}; \quad s(2) = 19.6 \text{ m}$$

From 0 s to 2 s:

$$v_{\text{avg}} = \frac{19.6 - 0}{2 - 0} = 9.8 \text{ m/s}$$

From 1 s to 2 s:

$$v_{\text{avg}} = \frac{19.6 - 4.9}{2 - 1} = 14.7 \text{ m/s}$$

Different intervals give different average velocities.
To find velocity at one instant, use smaller intervals around that instant.

Where this idea is useful

- Water depth in a reservoir.
- Height and velocity of a rocket.
- Changing values in economics.

APPROACHING INSTANTANEOUS VELOCITY

Keep one end of each interval at the instant of interest. Make the interval smaller from both sides.

$$s(t) = 4.9t^2; \quad t = 2 \text{ s}$$

Approaching from earlier times

t_1 (s)	$\frac{s(2) - s(t_1)}{2 - t_1}$ (m/s)
0	9.8
1	14.7
1.5	17.15
1.8	18.62
1.9	19.11
1.95	19.355
1.99	19.551

Approaching from later times

t_2 (s)	$\frac{s(t_2) - s(2)}{t_2 - 2}$ (m/s)
4	29.4
3	24.5
2.5	22.05
2.2	20.58
2.1	20.09
2.05	19.845
2.01	19.649

Both sets of average velocities approach the same value. This is the instantaneous velocity.

$$v(2) = 19.6 \text{ m/s}$$

FROM A SECANT TO A TANGENT

On a distance-time graph, the slope of a secant gives average velocity. As the nearby point approaches the fixed point, the secant approaches the tangent.

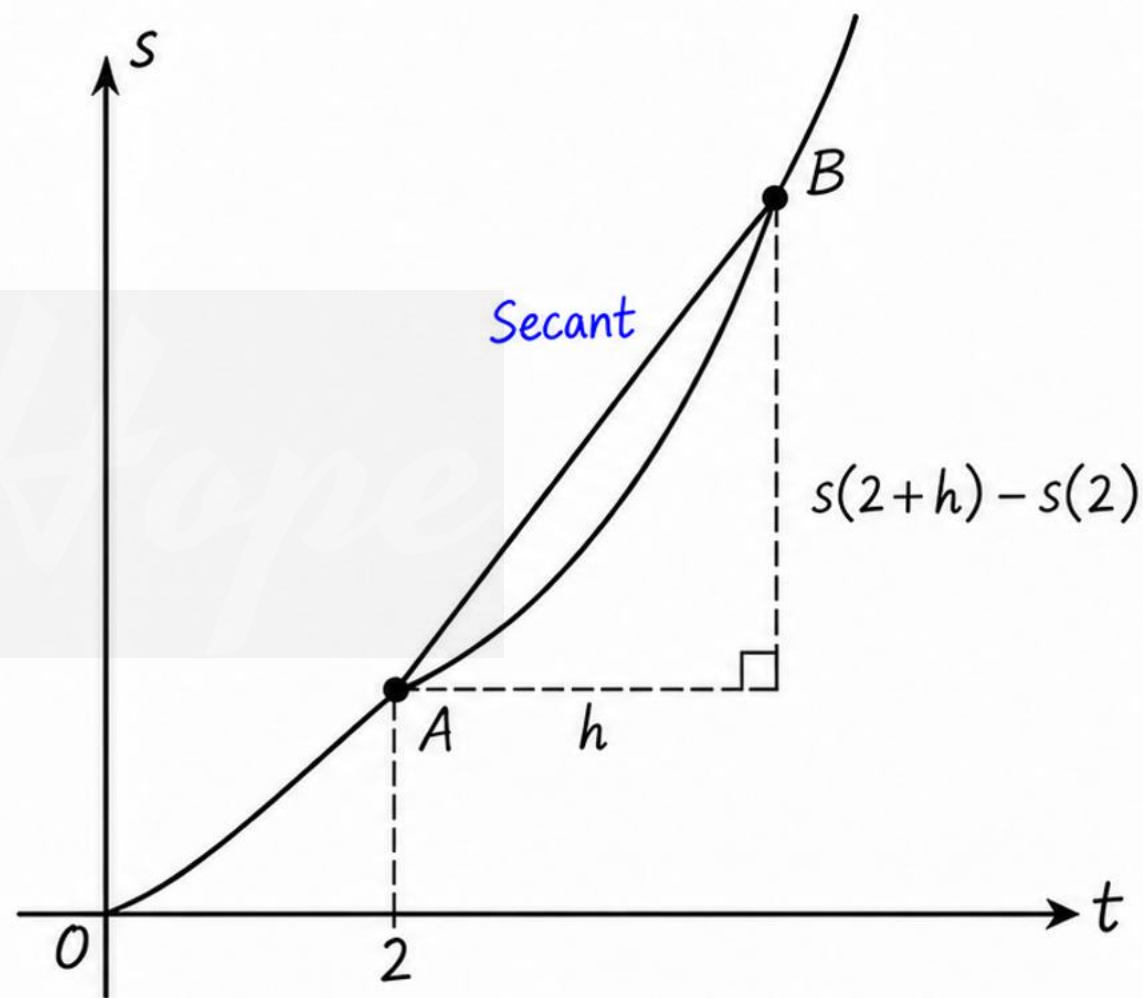
$$A = (2, s(2)); \quad B = (2+h, s(2+h))$$

$$h \neq 0$$

$$\begin{aligned}\text{Secant slope} &= \frac{s(2+h) - s(2)}{h} \\ &= \frac{4.9(2+h)^2 - 19.6}{h} \\ &= \frac{19.6h + 4.9h^2}{h} = 19.6 + 4.9h\end{aligned}$$

Let the time gap approach zero. The limiting slope gives velocity at the fixed instant.

$$\lim_{h \rightarrow 0} (19.6 + 4.9h) = 19.6 \text{ m/s}$$



WHAT DOES A LIMIT MEAN?

A limit tells us the value approached by a function when the input approaches a chosen point.
We use nearby inputs, not the point itself.

$$\lim_{x \rightarrow a} f(x) = \ell$$

Read this as: the limit of the function, as the input approaches the chosen point, is the approaching value.

A simple example

$$f(x) = x^2$$

x	-0.1	-0.01	0.01	0.1
x^2	0.01	0.0001	0.0001	0.01

$$\lim_{x \rightarrow 0} x^2 = 0$$

The point itself may be missing

$$g(x) = |x|, \quad x \neq 0$$

$$\lim_{x \rightarrow 0} |x| = 0$$

The absolute values approach zero from both sides, even when the function is not defined at the point.

A limit and the actual function value are different ideas.



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