

MATHEMATICS • CLASS 11

CHAPTER 2

Relations and Functions

CHAPTER NOTES



ORDERED PAIRS AND CARTESIAN PRODUCT

An ordered pair keeps the first and second objects in a fixed order.

$$(a, b) = (c, d) \text{ iff } a = c \text{ and } b = d.$$

Two ordered pairs are equal only when their first components are equal and their second components are equal.

Cartesian product

For any two sets P and Q , the Cartesian product $P \times Q$ is the set of all ordered pairs with the first element from P and the second element from Q .

$$P \times Q = \{ (p, q) : p \in P, q \in Q \}.$$

Example

Let the set of colours be

$$A = \{ \text{red, blue} \}.$$

Let the set of objects be

$$B = \{ b, c, s \},$$

where $b = \text{bag}$, $c = \text{coat}$, $s = \text{shirt}$.

The Cartesian product $A \times B$ is

$$A \times B = \{ (\text{red}, b), (\text{red}, c), (\text{red}, s), \\ (\text{blue}, b), (\text{blue}, c), (\text{blue}, s) \}.$$

Interpretation

Choose one colour first, then one object.

Reversing an ordered pair generally changes it.

$$(\text{red}, b) \neq (b, \text{red}).$$

COUNTING AND PROPERTIES OF PRODUCTS

Let $A = \{DL, MP, KA\}$ and $B = \{01, 02, 03\}$.

Choose a state code first from A and a number code second from B . Each ordered pair consists of a state code followed by a number code.

Elements of the product

The nine ordered pairs in $A \times B$ are

$(DL, 01)$ $(DL, 02)$ $(DL, 03)$

$(MP, 01)$ $(MP, 02)$ $(MP, 03)$

$(KA, 01)$ $(KA, 02)$ $(KA, 03)$

Number of elements

$$n(A) = 3, \quad n(B) = 3$$

$$n(A \times B) = 3 \times 3 = 9$$

Thus the product $A \times B$ has nine ordered pairs.

General result

If $n(A) = p$ and $n(B) = q$, then

$$n(A \times B) = pq$$

Products with the empty set

If one of the sets is empty, then the product is empty.

$$A \times \emptyset = \emptyset, \quad \emptyset \times B = \emptyset$$

If the first set is empty, there is no first choice possible. If the second set is empty, there is no second choice possible. Hence in both cases the product has no ordered pairs.

Infinite products

If BOTH sets are non-empty and at least one is infinite, then the Cartesian product is infinite.

Order matters

The order of the sets can change the ordered pairs.

$$(DL, 01) \neq (01, DL)$$

EQUAL AND REVERSED ORDERED PAIRS

Example 1

Given that the ordered pairs are equal,

$$(x + 1, y - 2) = (3, 1)$$

Equal ordered pairs have equal corresponding coordinates. Hence

$$x + 1 = 3 \quad \text{and} \quad y - 2 = 1$$

Subtract one from the first equation and add two to the second equation.

$$x = 2 \quad \text{and} \quad y = 3$$

Therefore, the values that satisfy the given equality are $x = 2$ and $y = 3$.

Example 2

Let $P = \{a, b, c\}$ and $Q = \{r\}$.

The Cartesian product $P \times Q$ is the set of all ordered pairs with first element from P and second element from Q . Thus

$$P \times Q = \{(a, r), (b, r), (c, r)\}$$

The Cartesian product $Q \times P$ is the set of all ordered pairs with first element from Q and second element from P . Thus

$$Q \times P = \{(r, a), (r, b), (r, c)\}$$

In each pair the first coordinate changes, for example $(a, r) \neq (r, a)$. Hence $P \times Q \neq Q \times P$, although both sets contain three ordered pairs.

PRODUCT OVER AN INTERSECTION

Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Compare two routes to find the same set of ordered pairs.

First route

Find the intersection of the second sets first.

$$B \cap C = \{4\}$$

Now take the Cartesian product with the first set.

$$\begin{aligned} A \times (B \cap C) &= A \times \{4\} \\ &= \{(1, 4), (2, 4), (3, 4)\} \end{aligned}$$

Since the only common element of the second sets is four, each element of the first set pairs only with four.

Second route

Find the Cartesian products separately.

$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$A \times C = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$$

Keep the ordered pairs that appear in both sets.

$$(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$$

The pairs common to both products are exactly those with second coordinate four, because four is the only element in both second sets.

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

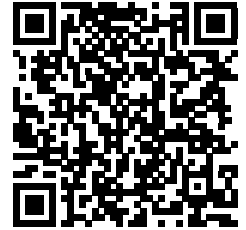


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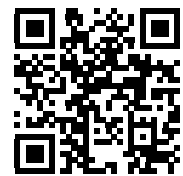


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