

MATHEMATICS • CLASS 11

CHAPTER 4

Complex Numbers and Quadratic Equations

CHAPTER NOTES



COMPLEX NUMBERS AND THEIR PARTS

The equation $x^2 + 1 = 0$ has no real solution, because for a real number x , the square x^2 is always nonnegative, so $x^2 + 1$ is strictly positive and cannot be zero.

The imaginary unit

We define the imaginary unit i by

$$i^2 = -1.$$

Complex numbers

A complex number is a number of the form

$$z = a + ib, \quad a, b \in \mathbb{R},$$

where a and b are real numbers.

Real and imaginary parts

For $z = a + ib$, the real part and imaginary part are

$$\operatorname{Re}(z) = a, \quad \operatorname{Im}(z) = b.$$

The imaginary part is the real number b itself, not the number ib .

Examples

For $z = 2 + 3i$, we have

$$\operatorname{Re}(z) = 2, \quad \operatorname{Im}(z) = 3.$$

For $z = -1 + \sqrt{3}i$, we have

$$\operatorname{Re}(z) = -1, \quad \operatorname{Im}(z) = \sqrt{3}.$$

Equality of complex numbers

For complex numbers $a + bi$ and $c + di$, we have

$$a + bi = c + di$$

if and only if $a = c$ and $b = d$.

Special cases

Real numbers are the complex numbers with $b = 0$, and purely imaginary numbers are the complex numbers with $a = 0$.

WORKED EXAMPLE: MATCHING REAL AND IMAGINARY PARTS

NCERT Example 1. Solve $4x + i(3x - y) = 3 - 6i$ with real x, y .

Comparing real parts

Equality of complex numbers implies that their real parts are equal.

$$4x = 3$$

$$x = \frac{3}{4}$$

Comparing imaginary parts

Equality of complex numbers also implies that their imaginary parts are equal.

$$3x - y = -6$$

$$y = 3x + 6$$

Finding the value of y

Substitute the value of x into the expression for y .

$$\begin{aligned} y &= 3x + 6 \\ &= 3\left(\frac{3}{4}\right) + 6 \\ &= \frac{9}{4} + \frac{24}{4} \\ &= \frac{33}{4} \end{aligned}$$

Conclusion

Equality of complex numbers means that both the real part equation and the imaginary part equation hold. Hence,

$$x = \frac{3}{4}, \quad y = \frac{33}{4}.$$

ADDITION AND SUBTRACTION OF COMPLEX NUMBERS

Let $z_1 = a + bi$ and $z_2 = c + di$ be two complex numbers, where a, b, c, d are real numbers.

Then

$$z_1 + z_2 = (a + c) + (b + d)i$$

$$z_1 - z_2 = (a - c) + (b - d)i$$

Add like parts, that is, real parts together and imaginary parts together.

Worked example

Add the following complex numbers.

$$\begin{aligned}(2 + 3i) + (-6 + 5i) &= (2 - 6) + (3 + 5)i \\ &= -4 + 8i\end{aligned}$$

Subtract the following complex numbers.

$$\begin{aligned}(6 + 3i) - (2 - i) &= (6 - 2) + (3 - (-1))i \\ &= 4 + 4i\end{aligned}$$

Closure

If z_1 and z_2 are complex numbers, then $z_1 + z_2$ and $z_1 - z_2$ are also complex numbers. Hence the set of all complex numbers is closed under addition and subtraction.

Commutativity of addition

For any complex numbers z_1 and z_2 ,

$$z_1 + z_2 = z_2 + z_1.$$

Associativity of addition

For any complex numbers z_1, z_2 and z_3 ,

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3).$$

Additive identity

The complex number $0 = 0 + 0i$ is the additive identity, since for any complex number z ,

$$z + 0 = 0 + z = z.$$

Additive inverse

For any complex number $z = a + bi$, the additive inverse is

$$-z = -(a + bi) = -a - bi,$$

since $z + (-z) = 0$.

MULTIPLICATION OF COMPLEX NUMBERS

Let $z_1 = a + bi$ and $z_2 = c + di$, where a, b, c, d are real numbers. Using $i^2 = -1$, we derive the product $z_1 z_2$ in terms of real and imaginary parts.

$$\begin{aligned} z_1 z_2 &= (a + bi)(c + di) \\ &= ac + adi + bci + bdi^2 \\ &= ac + adi + bci - bd \\ &= (ac - bd) + (ad + bc)i \end{aligned}$$

Example

Find the product $(3 + 5i)(2 + 6i)$.

$$\begin{aligned} (3 + 5i)(2 + 6i) &= 3 \times 2 + 3 \times 6i + 5i \times 2 + 5i \times 6i \\ &= 6 + 18i + 10i + 30i^2 \\ &= 6 + 28i + 30(-1) \\ &= 6 + 28i - 30 = -24 + 28i \end{aligned}$$

Closure

If z_1 and z_2 are complex numbers, then $z_1 z_2$ is also a complex number of the form $(ac - bd) + (ad + bc)i$, where $ac - bd$ and $ad + bc$ are real numbers.

Commutativity

For complex numbers z_1 and z_2 ,

$$z_1 z_2 = z_2 z_1.$$

Associativity

For complex numbers z_1 , z_2 and z_3 ,

$$(z_1 z_2) z_3 = z_1 (z_2 z_3).$$

Distributivity over addition

For complex numbers z_1 , z_2 and z_3 ,

$$z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3.$$

Identity, zero and one

The multiplicative identity is 1, where $1 = 1 + 0i$.

The zero is 0, where $0 = 0 + 0i$.

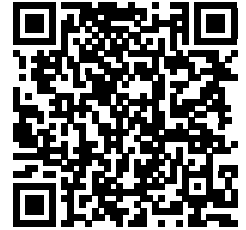


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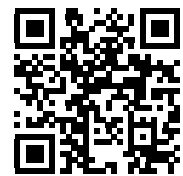


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