

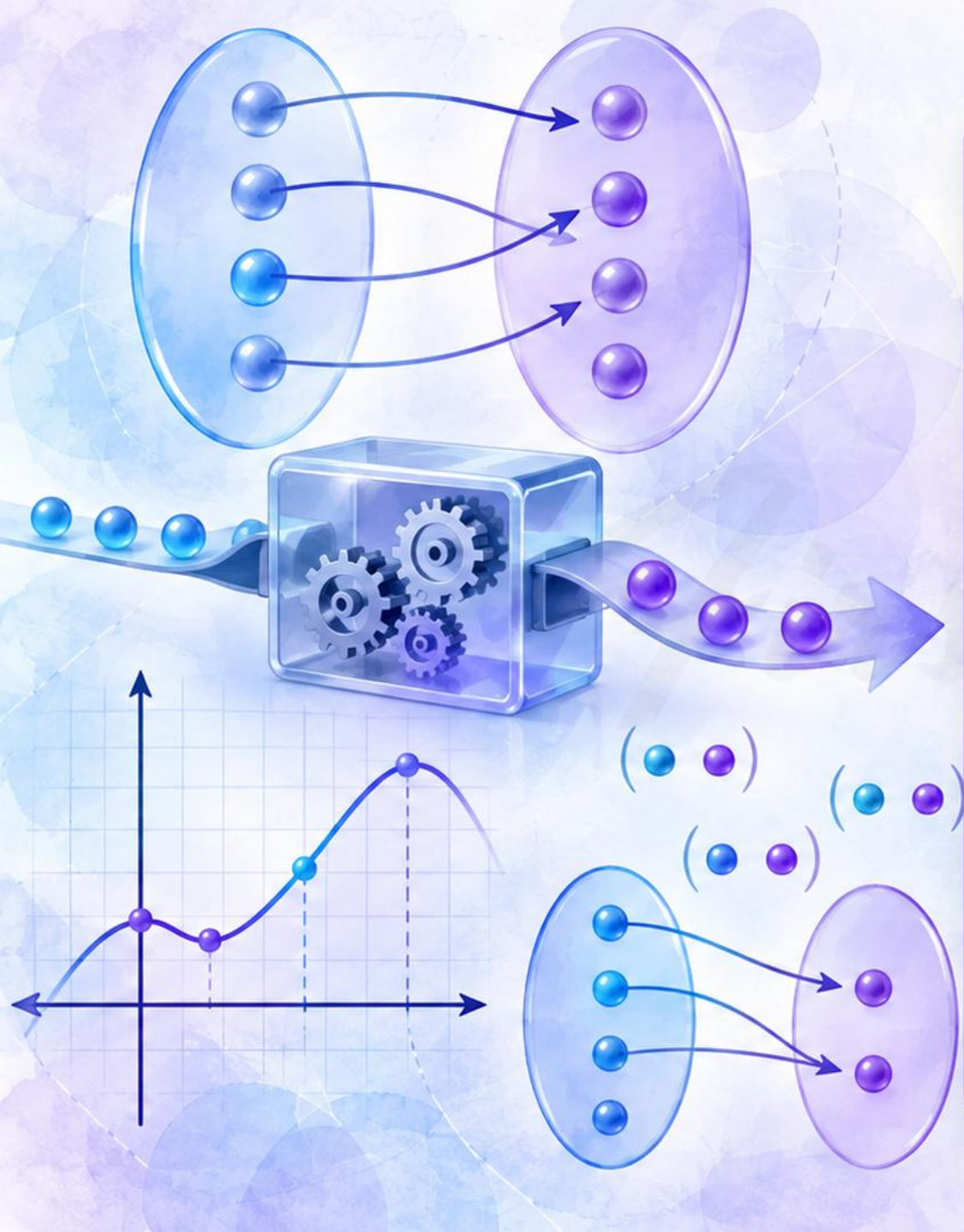
MATHEMATICS • CLASS 12

1

CHAPTER

Relations and Functions

NCERT SOLUTIONS



RELATION $y = 3x$ — EXERCISE 1.1 Q1(i)

Q. In $A = \{1, 2, 3, \dots, 14\}$, $R = \{(x, y) : 3x - y = 0\}$. Determine whether R is reflexive, symmetric and transitive.

Answer: Since $3x - y = 0$, we have $y = 3x$. Both x and y must belong to $A = \{1, 2, 3, \dots, 14\}$. For $x \in A$, the value $3x$ is in A only for $x = 1, 2, 3, 4$ (since $3 \times 5 = 15 > 14$). Hence $R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$.

Reflexive: A relation R on A is reflexive if $(x, x) \in R$ for every $x \in A$. For $x = 1$, $(1, 1)$ would be in R , but $3(1) - 1 = 2 \neq 0$. So $(1, 1) \notin R$. Hence R is not reflexive.

Symmetric: A relation R is symmetric if $(x, y) \in R$ implies $(y, x) \in R$. We have $(1, 3) \in R$, but $(3, 1) \notin R$ since $3(3) - 1 = 8 \neq 0$. Hence R is not symmetric.

Transitive: A relation R is transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$. We have $(1, 3) \in R$ and $(3, 9) \in R$, but $(1, 9) \notin R$ since $3(1) - 9 = -6 \neq 0$. Hence R is not transitive.

Therefore, R is neither reflexive nor symmetric nor transitive.

NATURAL-NUMBER RELATION — EXERCISE 1.1 Q1(ii)

Q. In N , $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$. Determine whether R is reflexive, symmetric and transitive.

Answer: Since $x < 4$ and $x \in N = \{1, 2, 3, \dots\}$, we have $x = 1, 2, 3$.

Then $y = x + 5$ gives the ordered pairs

$$R = \{(1, 6), (2, 7), (3, 8)\}.$$

Reflexive: A relation R on N is reflexive if $(x, x) \in R$ for every $x \in N$.

Here $(1, 1) \notin R$ (since $R = \{(1, 6), (2, 7), (3, 8)\}$).

Hence R is not reflexive.

Symmetric: A relation R on N is symmetric if $(x, y) \in R$ implies $(y, x) \in R$.

We have $(1, 6) \in R$, but $(6, 1) \notin R$ (no ordered pair in R has first component 6).

Hence R is not symmetric.

Transitive: A relation R on N is transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$.

In R , the first components are 1, 2 and 3, while the second components are 6, 7 and 8.

A pair (y, z) with $y = 6, 7$ or 8 cannot occur in R because every first component must be less than 4. Hence no two pairs $(x, y), (y, z)$ in R can form a chain.

Therefore, the implication for transitivity is true (vacuously), and R is transitive.

Thus, R is transitive, but neither reflexive nor symmetric.

DIVISIBILITY RELATION – EXERCISE 1.1 Q1(iii)

Q. In $A = \{1, 2, 3, 4, 5, 6\}$, $R = \{(x, y) : y \text{ is divisible by } x\}$. Determine whether R is reflexive, symmetric and transitive.

Answer:

Reflexive: A relation R on A is reflexive if $(x, x) \in R$ for every $x \in A$.

For any $x \in A$, we have $x = x \cdot 1$. Hence x divides x , so $(x, x) \in R$.

Therefore, R is reflexive.

Symmetric: A relation R on A is symmetric if $(x, y) \in R$ implies $(y, x) \in R$.

Now 1 divides 2, so $(1, 2) \in R$. But 2 does not divide 1, so $(2, 1) \notin R$.

Hence, there exists $(1, 2) \in R$ such that $(2, 1) \notin R$.

Therefore, R is not symmetric.

Transitive: A relation R on A is transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$.

Let $(x, y) \in R$ and $(y, z) \in R$. Then y is divisible by x and z is divisible by y .

So, there exist $m, n \in \mathbb{N}$ such that $y = xm$ and $z = yn$.

Substituting, we get $z = yn = (xm)n = x(mn)$. Hence z is divisible by x , i.e. $(x, z) \in R$.

Therefore, R is transitive.

Thus, R is reflexive and transitive, but not symmetric.

INTEGER-DIFFERENCE RELATION — EXERCISE 1.1 Q1 (iv)

Q. In $\mathbb{Z}$, $R = \{(x, y) : x - y \text{ is an integer}\}$. Determine whether R is reflexive, symmetric and transitive.

Answer: For any integers x and y , $x - y$ is automatically an integer.
Hence every ordered pair $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ satisfies $x - y \in \mathbb{Z}$.
Therefore, $R = \{(x, y) : x - y \text{ is an integer}\} = \mathbb{Z} \times \mathbb{Z}$, the universal relation on $\mathbb{Z}$.

Reflexive: A relation R on $\mathbb{Z}$ is reflexive if $(x, x) \in R$ for every $x \in \mathbb{Z}$.
For any integer x , $x - x = 0$, and $0 \in \mathbb{Z}$.
Hence $(x, x) \in R$ for every $x \in \mathbb{Z}$. Therefore, R is reflexive.

Symmetric: A relation R on $\mathbb{Z}$ is symmetric if $(x, y) \in R$ implies $(y, x) \in R$.
Let $(x, y) \in R$. Then $x - y \in \mathbb{Z}$. Now $y - x = -(x - y)$.
Since $x - y \in \mathbb{Z} \Rightarrow -(x - y) \in \mathbb{Z}$, we have $y - x \in \mathbb{Z}$, i.e. $(y, x) \in R$.
Hence, R is symmetric.

Transitive: A relation R on $\mathbb{Z}$ is transitive if $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$.
Let $(x, y) \in R$ and $(y, z) \in R$. Then $x - y \in \mathbb{Z}$ and $y - z \in \mathbb{Z}$.
Now $x - z = (x - y) + (y - z)$. Since the sum of two integers is an integer, $x - z \in \mathbb{Z}$.
Hence, $(x, z) \in R$. Therefore, R is transitive.

R is reflexive, symmetric and transitive; hence it is an equivalence relation.

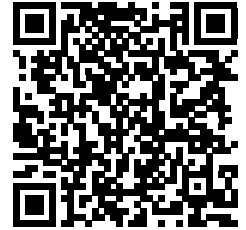


FREE PREVIEW

[Click here to visit our website](#)

SCHOOL NOTES & NCERT SOLUTIONS

Complete notes and
NCERT solutions are
available on
FirstHope app only.



[Scan or click here](#)

Click here to get the app



Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram



[Scan or click here](#)

@FirstHope_CBSE_Notes

Instagram

@firsthope.school

[Click here to follow >](#)

YouTube channel

@firsthope.school

[Click here to watch >](#)

FirstHope website

firsthope.co.in

[Click here to visit our website >](#)

CBSE / Class 9 / Class 10 / Class 11 / Class 12

Preview notes and NCERT solutions. Full PDFs in the FirstHope app.