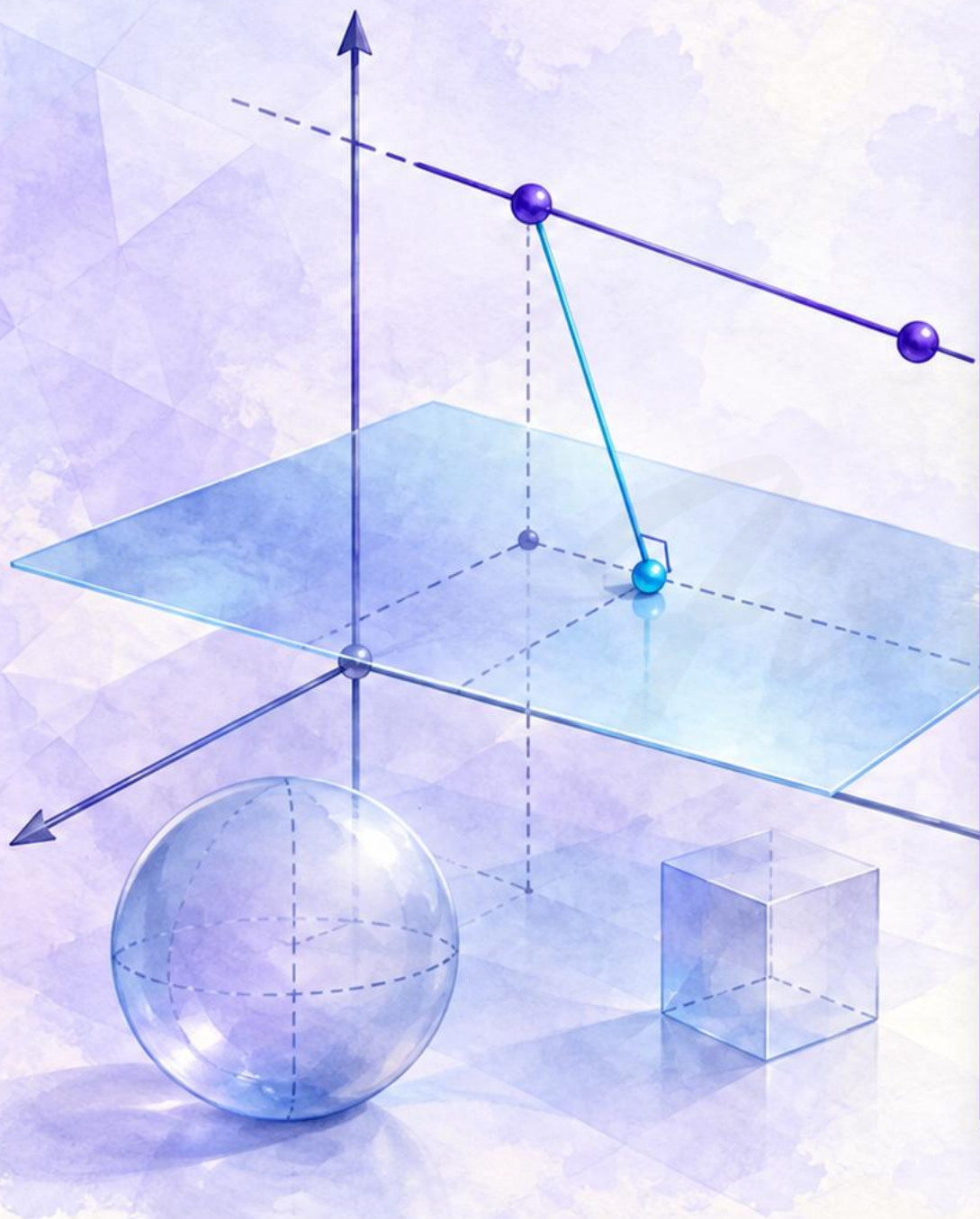




MATHEMATICS • CLASS 12

11

CHAPTER

Three Dimensional Geometry

NCERT SOLUTIONS

DIRECTION COSINES — EXERCISE 11.1 Q1

Q. If a line makes angles 90° , 135° , 45° with the x , y and z -axes respectively, find its direction cosines.

Answer:

Let a line make angles α , β , γ with the x , y and z -axes respectively. The direction cosines of the line are $\cos \alpha$, $\cos \beta$, $\cos \gamma$.

In this question, the given angles are $\alpha = 90^\circ$, $\beta = 135^\circ$, $\gamma = 45^\circ$.

Therefore, the direction cosines are

$$l = \cos 90^\circ = 0,$$

$$m = \cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}},$$

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

Hence, $(0, -1/\sqrt{2}, 1/\sqrt{2})$.

DIRECTION COSINES — EXERCISE 11.1 Q2

Q. Find the direction cosines of a line which makes equal angles with the coordinate axes.

Answer:

Let the direction cosines of the required line be l, m, n .

Since the line makes equal angles with the coordinate axes,

$$l = m = n.$$

Also, the direction cosines satisfy

$$l^2 + m^2 + n^2 = 1.$$

Substituting $l = m = n$, we get

$$3l^2 = 1 \Rightarrow l^2 = \frac{1}{3} \Rightarrow l = \pm \frac{1}{\sqrt{3}}.$$

Hence, the direction cosines are either all $1/\sqrt{3}$ or all $-1/\sqrt{3}$, corresponding to the two opposite directions of the same line.

Hence, $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ or $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$.

DIRECTION COSINES — EXERCISE 11.1 Q3

Q. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

Answer:

Let the direction ratios of the line be a, b, c . Then its direction cosines (l, m, n) are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

Here the direction ratios are $-18, 12, -4$, so $a = -18, b = 12, c = -4$.

First find the magnitude of the direction vector:

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{(-18)^2 + 12^2 + (-4)^2} = \sqrt{324 + 144 + 16} = \sqrt{484} = 22.$$

Now divide each direction ratio by 22 to obtain the direction cosines:

$$l = \frac{-18}{22} = -\frac{9}{11}, \quad m = \frac{12}{22} = \frac{6}{11}, \quad n = \frac{-4}{22} = -\frac{2}{11}.$$

Hence, $(-9/11, 6/11, -2/11)$.

DIRECTION COSINES — EXERCISE 11.1 Q4

Q. Show that the points $(2, 3, 4)$, $(-1, -2, 1)$ and $(5, 8, 7)$ are collinear.

Answer:

To show that three points are collinear, it is sufficient to show that the direction vectors of the lines joining one point to the other two points are proportional (i.e., parallel).

Let $A(2, 3, 4)$, $B(-1, -2, 1)$ and $C(5, 8, 7)$.

Direction ratios of the line joining A and B are

$$\overrightarrow{AB} = (-1 - 2, -2 - 3, 1 - 4) = (-3, -5, -3).$$

Direction ratios of the line joining A and C are

$$\overrightarrow{AC} = (5 - 2, 8 - 3, 7 - 4) = (3, 5, 3).$$

We observe that $\overrightarrow{AB} = (-3, -5, -3) = -(3, 5, 3) = -\overrightarrow{AC}$.

Thus, the direction ratios of AB and AC are proportional (one is a negative multiple of the other), so AB is parallel to AC.

Since both AB and AC pass through the common point A, the three points A, B, C lie on the same straight line.

Hence, the points $(2, 3, 4)$, $(-1, -2, 1)$ and $(5, 8, 7)$ are collinear.



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