



MATHEMATICS • CLASS 12

6

CHAPTER

Applications of Derivatives

CHAPTER NOTES

RATES OF CHANGE

Meaning and sign

A derivative tells how fast one quantity changes with another.

At a particular input, it gives the instantaneous rate.

$y = f(x)$, $\frac{dy}{dx} = f'(x)$; at $x = x_0$ the rate is $f'(x_0)$.

When the independent variable is time, the derivative is a rate per unit time.

$$v = \frac{ds}{dt}$$

A positive rate means increase; a negative rate means decrease.

General rule

If both quantities depend on a common parameter, use the chain rule.

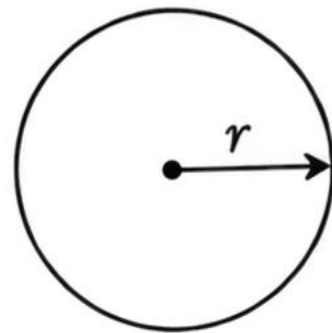
$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}; \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0.$$

Area with respect to radius

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$r = 5 \text{ cm} \Rightarrow \frac{dA}{dr} = 10\pi \text{ cm}^2/\text{cm}.$$

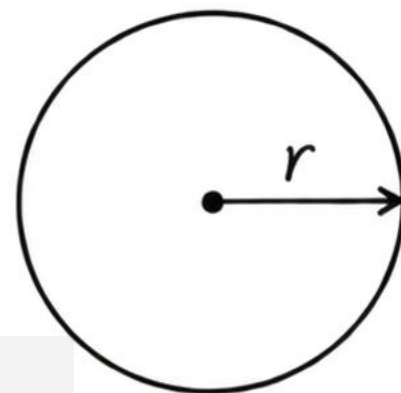


This rate is per unit radius, not per second.
A time rate needs the rate of change of radius.

CHANGING CIRCULAR AREAS

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$



Differentiate with respect to time and multiply by the rate of change of radius.

Circular waves

The radius grows at the given speed.

$$\frac{dr}{dt} = 4 \text{ cm/s}, \quad r = 10 \text{ cm}$$

$$\frac{dA}{dt} = 2\pi(10)(4) = 80\pi \text{ cm}^2/\text{s}$$

An expanding disc

Heating increases the radius at a constant rate.

$$\frac{dr}{dt} = 0.05 \text{ cm/s}, \quad r = 3.2 \text{ cm}$$

$$\frac{dA}{dt} = 2\pi(3.2)(0.05) = 0.32\pi \text{ cm}^2/\text{s}$$

Both areas are increasing. The unit is area per second.

RELATED RATES: A CUBE

Given

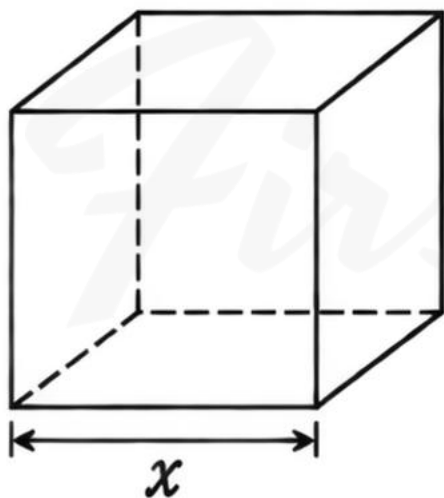
The cube's volume increases at the given rate.
Find the rate of increase of its total surface area
at the stated edge length.

$$V = x^3$$

$$S = 6x^2$$

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{s}$$

$$x = 10 \text{ cm}$$



Solution

Relate the edge rate to the volume rate

First connect the edge rate to the volume rate.

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{9}{3x^2} = \frac{3}{x^2}$$

Use the edge rate in the surface-area derivative

Then use the edge rate in the surface-area derivative.

$$\frac{dS}{dt} = 12x \frac{dx}{dt} = 12x \left(\frac{3}{x^2} \right) = \frac{36}{x}$$

Substitute the edge length

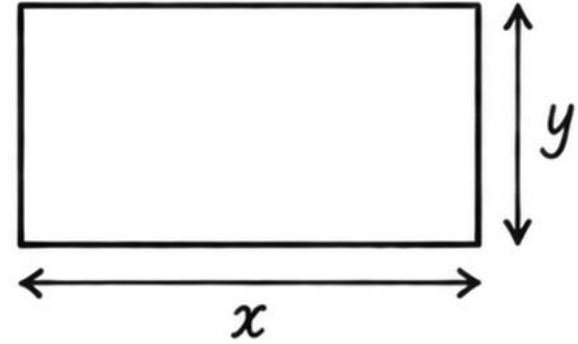
Substitute the edge length after differentiation.

$$x = 10 \Rightarrow \frac{dS}{dt} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{s}$$

RELATED RATES: A RECTANGLE

Given information

- The length decreases while the width increases.
- The negative sign is essential for a decreasing quantity.
- $\frac{dx}{dt} = -3 \text{ cm/min}$; $\frac{dy}{dt} = 2 \text{ cm/min}$; $x = 10 \text{ cm}$, $y = 6 \text{ cm}$.



Perimeter

The perimeter of a rectangle is

$$P = 2(x + y).$$

Differentiating with respect to time gives

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = 2(-3 + 2) = -2 \text{ cm/min}.$$

The perimeter decreases.

Area

The area of a rectangle is

$$A = xy.$$

Differentiating with respect to time gives

$$\frac{dA}{dt} = y \frac{dx}{dt} + x \frac{dy}{dt}.$$

$$\frac{dA}{dt} = 6(-3) + 10(2) = 2 \text{ cm}^2/\text{min}.$$

The area increases even though the length decreases.



FREE PREVIEW

[Click here to visit our website](#)

SCHOOL NOTES & NCERT SOLUTIONS

Complete notes and
NCERT solutions are
available on
FirstHope app only.



[Scan or click here](#)

Click here to get the app

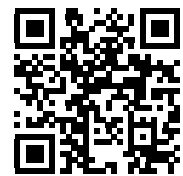


Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram



[Scan or click here](#)

@FirstHope_CBSE_Notes

Instagram

@firsthope.school

[Click here to follow >](#)

YouTube channel

@firsthope.school

[Click here to watch >](#)

FirstHope website

firsthope.co.in

[Click here to visit our website >](#)

CBSE / Class 9 / Class 10 / Class 11 / Class 12

Preview notes and NCERT solutions. Full PDFs in the FirstHope app.