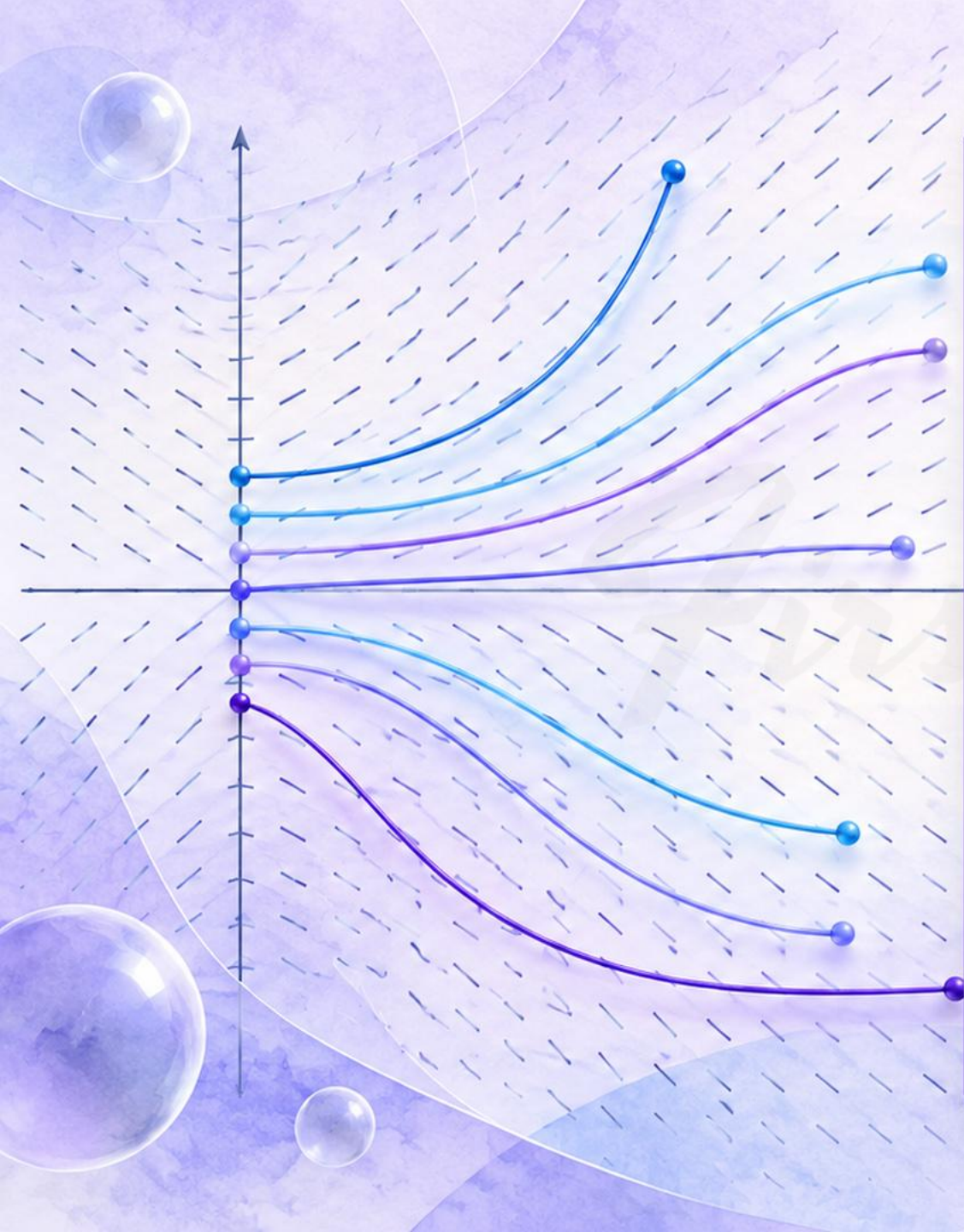




MATHEMATICS • CLASS 12

9

CHAPTER

Differential Equations

CHAPTER NOTES

DIFFERENTIAL EQUATIONS & ORDER

A differential equation is an equation involving derivatives of a dependent variable with respect to an independent variable.

$y = f(x)$: x is independent; y is dependent.

An ordinary differential equation involves derivatives with respect to one independent variable. This chapter studies ordinary differential equations.

Derivative notation

$$y' = \frac{dy}{dx} ; \quad y'' = \frac{d^2y}{dx^2} ; \quad y''' = \frac{d^3y}{dx^3} ; \quad y^{(n)} = \frac{d^ny}{dx^n} .$$

Order

The order is the order of the highest derivative present.

$$y' = e^x \rightarrow \text{order 1} \quad y'' + y = 0 \rightarrow \text{order 2} \quad y''' + x^2(y'')^3 = 0 \rightarrow \text{order 3} .$$

Look at the derivative order, not its power.

DEGREE OF A DIFFERENTIAL EQUATION

Degree is defined only when the equation is a polynomial in its derivatives.

The degree is the power of the highest-order derivative in that polynomial form.

The coefficients may contain the variables or functions of them.

$$y''' + 2(y'')^2 - y' + y = 0 : \text{ order} = 3, \text{ degree} = 1.$$

$$(y')^2 + y' - \sin^2 y = 0 : \text{ order} = 1, \text{ degree} = 2.$$

A trigonometric function of the dependent variable does not prevent a polynomial in the derivatives.

$$y' + \sin(y') = 0 : \text{ order} = 1, \text{ degree not defined.}$$

Here a derivative occurs inside a sine function, so the equation is not polynomial in its derivatives.

Order and degree, when defined, are positive integers.

ORDER & DEGREE: WORKED CASES

$$y' - \cos x = 0$$

The highest derivative is the first derivative, raised to the first power.

order = 1; degree = 1.

$$xyy'' + x(y')^2 - yy' = 0$$

The highest derivative is the second derivative. Its power is one, even though the first derivative is squared.

order = 2; degree = 1.

$$y''' + y'' + e^{y'} = 0$$

The third derivative sets the order. A derivative occurs in the exponent, so the equation is not polynomial in derivatives.

order = 3; degree not defined.

GENERAL & PARTICULAR SOLUTIONS

A solution is a function that satisfies the differential equation after substitution. Its graph is called an integral curve.

General solution

A general solution, or primitive, contains arbitrary constants. In this chapter, their number equals the order of the differential equation.

$$y = a \sin(x + b), \quad a, b \in \mathbb{R}$$

$$y' = a \cos(x + b); \quad y'' = -a \sin(x + b) = -y$$

$$y'' + y = 0$$

Particular solution

Fixing the arbitrary constants gives a particular solution. It has no arbitrary constants left.

$$a = 2, \quad b = \frac{\pi}{4}$$

$$y = 2 \sin\left(x + \frac{\pi}{4}\right)$$

Extra conditions select a particular member of the solution family.



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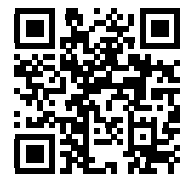


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