

MATHEMATICS • CLASS 9

CHAPTER 8

Predicting What Comes Next

Exploring Sequences and Progressions

NCERT SOLUTIONS



SEQUENCES — PATTERNS THAT CONTINUE

1) Definition

- Sequence: an ordered list of numbers. Each number is called a term.
- The position matters: first term, second term, ...
- The dots ... mean the pattern continues.

$a_1, a_2, a_3, a_4, a_5, \dots$
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 1st 2nd 3rd 4th 5th ...
 positions

2) Think-and-Reflect: Find the rule and the next three terms.

① Natural numbers → 1, 2, 3, 4, 5, 6, ...

+1 +1 +1 +1 +1

Rule: add 1
(each term is 1 more than the previous.)

• • • • •

Next three terms:
7, 8, 9

② Odd numbers → 1, 3, 5, 7, 9, 11, ...

+2 +2 +2 +2 +2

Rule: add 2
(each term is 2 more than the previous.)

• • • • •

Next three terms:
13, 15, 17

③ Triangular numbers → 1, 3, 6, 10, 15, 21, ...

+2 +3 +4 +5 +6

Differences: +2, +3, +4, +5, +6
↓
Next differences: +7, +8, +9

1 3 6 10 15 21 ...

Next three terms:
28, 36, 45

④ Square numbers → 1, 4, 9, 16, 25, 36, ...

■ ■■ ■■■ ■■■■ ■■■■■

Rule: terms are n^2
($1^2, 2^2, 3^2, 4^2, 5^2, 6^2, \dots$)

n^{th} term = n^2 ,
where $n = 1, 2, 3, \dots$

Next three terms:
49, 64, 81

3) Finite vs Infinite sequences

- Infinite sequence: continues without end.
Example: 1, 2, 3, ... (keeps going forever)

These help us understand patterns better!

- Finite sequence: has a fixed number of terms.
Example: 6, 12, 24, 48, 96, has 5 terms.

4) Remember

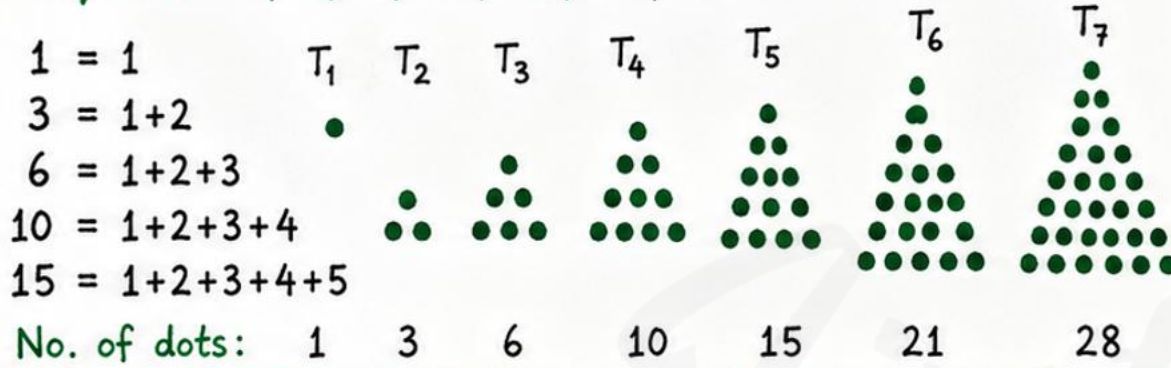
★ You can predict later terms only after identifying the rule.

★ Different sequences may begin similarly but follow different rules.

TRIANGULAR & SQUARE NUMBER PATTERNS

TRIANGULAR NUMBERS

Sequence: 1, 3, 6, 10, 15, 21, ...



Therefore n^{th} triangular number:

$$T_n = 1+2+\dots+n = \frac{n(n+1)}{2}$$

Asked pattern extension :

$$T_6 = 21$$

$$T_7 = 21+7 = 28$$

Hence, the next two triangular numbers are 21 and 28 (after the first five shown).

SQUARE NUMBERS AS SUMS OF ODD NUMBERS

Sequence: 1, 4, 9, 16, 25, 36, ...

Differences: +3, +5, +7, +9, +11

$$1^2 = 1$$

$$2^2 = 1+3 = 4$$

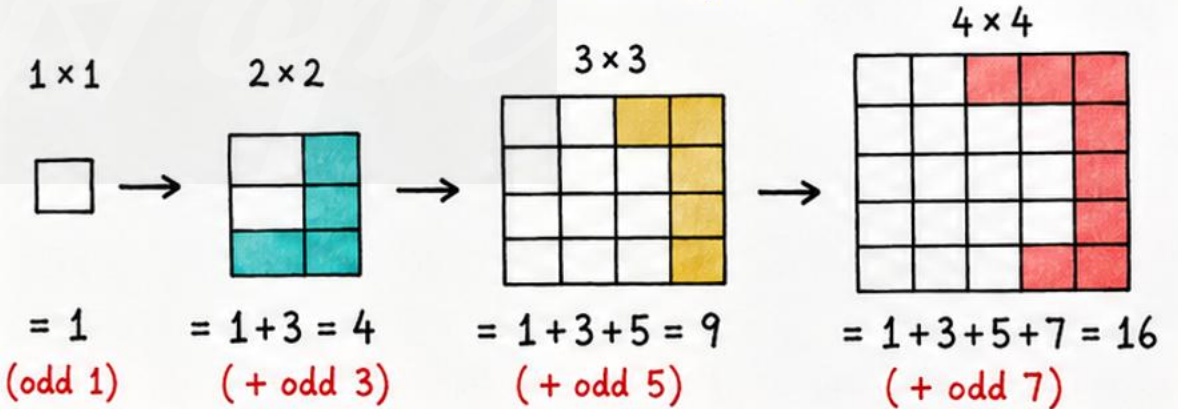
$$3^2 = 1+3+5 = 9$$

$$4^2 = 1+3+5+7 = 16$$

General relationship:

$$n^2 = 1+3+5+\dots+(2n-1)$$

Adding the next odd-number border turns an $(n-1) \times (n-1)$ square into an $n \times n$ square.



Triangular number $\rightarrow$ sum of first n natural numbers.

$$T_n = \frac{n(n+1)}{2}$$

Square number $\rightarrow$ sum of first n odd numbers.

$$n^2 = 1+3+5+\dots+(2n-1)$$

Changing differences show why these are not APs, even though they follow clear patterns.

PATTERN EXERCISE — TERMS & RUNNING SUMS

① Identify rule:

Consecutive difference = +3

Original sequence:

1, 4, 7, 10, 13, 16, 19, 22, 25, 28, ...

Next four terms after 13:

16, 19, 22, 25

Answer: 16, 19, 22, 25

② Define running sum simply:

At position n , add the first n terms of the original sequence.

③ First 10 terms of the running-sum sequence:

1, 5, 12, 22, 35, 51, 70, 92, 117, 145

④ Check 1: Original n th term

$$t_n = 1 + (n-1) \times 3 = 3n - 2$$

Check $t_{10} = 3(10) - 2 = \boxed{28}$

⑤ Check 2: Check final running sum by pairing original terms

$$(1+28) + (4+25) + (7+22) + (10+19) + (13+16)$$

$$= 29 \times 5 = \boxed{145}$$



Do not confuse the original terms with the running-sum terms.



Running sum means the total so far.

n	original term	running addition	running sum
1	1	1	1
2	4	$1 + 4$	5
3	7	$1 + 4 + 7$	12
4	10	$12 + 10$	22
5	13	$22 + 13$	35
6	16	$35 + 16$	51
7	19	$51 + 19$	70
8	22	$70 + 22$	92
9	25	$92 + 25$	117
10	28	$117 + 28$	145

TERM NOTATION & DIFFERENT KINDS OF SEQUENCES

1 Term notation:

t_1 = first term, t_2 = second term, ..., t_n = term in position n .

Example (odd numbers): $t_1 = 1$, $t_2 = 3$, $t_3 = 5$, $t_4 = 7$.

- Subscript tells the position; it is not multiplication.
- Different letters t_n , s_n , u_n may name different sequences.

2 Exercise solution — Triangular numbers

Sequence: 1, 3, 6, 10, 15, 21, 28, 36, ...

$$t_5 = 15$$

$$t_6 = 21$$

$$t_7 = 28$$

$$t_8 = 36$$



Answer:

(15, 21, 28, 36)

3 Important note:

The position n is a natural-number index 1, 2, 3, ... ; the term value itself may be positive, zero, negative, fractional or any real number.

4 Think-and-Reflect: Five types/examples

- ① Increasing arithmetic: $-7, -3, 1, 5, 9, \dots$ → Add 4 each time.
- ② Decreasing unit fractions: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$ → Each term is reciprocal of counting numbers.
- ③ Alternating: $1, -1, 1, -1, \dots$ → Signs alternate +, -.
- ④ Growing powers: $6, 12, 24, 48, 96, \dots$ → Multiply by 2 each time.
(This can be made infinite by writing an ellipsis ...)
- ⑤ Finite sequence: $6, 12, 24, 48, 96$ → Stops after 5 terms.

5 Bottom comparison:

- Term = value.
- Position/subscript = where the value occurs.
- The same value can appear at more than one position in a repeating sequence.



FREE PREVIEW

[Click here to visit our website](#)

SCHOOL NOTES & NCERT SOLUTIONS

Complete notes and
NCERT solutions are
available on
FirstHope app only.



[Scan or click here](#)

Click here to get the app



Available on Google Play

JOIN THE FIRSTHOPE COMMUNITY

Join our Telegram channel

Click here to join Telegram



[Scan or click here](#)

@FirstHope_CBSE_Notes

Instagram

@firsthope.school

[Click here to follow >](#)

YouTube channel

@firsthope.school

[Click here to watch >](#)

FirstHope website

firsthope.co.in

[Click here to visit our website >](#)

CBSE / Class 9 / Class 10 / Class 11 / Class 12

Preview notes and NCERT solutions. Full PDFs in the FirstHope app.